Solitary wave solutions to the general time fractional Riccati equation and Hirota-Satsuma equations

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Abstract: In the present work, we set up new careful answers for the Hirota-Satsuma conditions. We used to build voyaging wave arrangements regarding an exaggerated digression works by tanh. New groups of lone wave arrangements and occasional arrangements are additionally gotten for Hirota-Satsuma conditions. Our methodology is diminishing the size of the computational received in different procedures with no conditions to apply on any arrangement of halfway differential conditions. The Maple programming is utilized all through for the arrangement of the arrangement of arithmetical conditions got en route and furthermore for the graphical delineations, separately. We found that for Hirota-Satsuma conditions when the estimation of memory list (time-fragmentary request) is near nothing, the arrangements independent and produce a wave-like design.

Keywords: Travelling wave solutions, Homogeneous balance, Hyperbolic function solution, conformable fractional derivative, Hirota-Satsuma equations.

1. Introduction:

Nonlinear partial differential equations (NPDEs) have an essential vicinity in implemented arithmetic and physics. Differential equations featuring fractional orders are of fantastic importance in lots of sciences and engineering fields given that they first-rate describe bodily conditions. These equations are mathematical fashions of bodily phenomenon that stand up in engineering, chemistry, biology, mechanics and physics. It's miles very crucial to have information approximately the answers of mathematical fashions. to higher Recognize the mechanisms of the mathematical fashions, it's far important to remedy those equations. Thus, it has a crucial region to gain the analytic solutions of nonlinear differential equations in implemented sciences. These days, it has end up appealing solving those equations. Consequently, a few techniques had been developed through sciences. some of them are: Homogeneous balance method [1], Hirota method [2], Backlund transformation [3], Burger's KdV method [4], Enhanced $\left(\frac{G'}{G}\right)$-expansion method [5, 6], $\left(\frac{G'}{G}\right)$-expansion method [7, 9, 10], the exp(-Φ(ξ))-expansion method [11], $\left(\frac{G'}{G},\frac{1}{G}\right)$-expansion method [8], the tan $(F(\xi)/2)$-expansion method [12], modified generalized Riccati equation method [7]. As of late, [17] saw that the arrangements of the changed Hirota-Satsuma framework bifurcate and produce a wave design full memory and the example disappears.

While, various researchers utilized the fragmentary auxiliary [13, 14] and other prominent fractional auxiliaries nearby some intelligent methodologies to handle various solitary wave issues by time fractional equations[15], for instance, the Hirota-Satsuma conditions [16].

Moreover, numerous specialists used this fragmentary subordinate and other notable partial subsidiaries close by some logical strategies to take care of numerous singular wave issues, for example, the Hirota-Satsuma conditions [16].

$$u_t + u_{xxx} + 6uu_x - 6vv_x = 0$$
$$v_t - 2v_{xxx} - 6uv_x = 0 \tag{1}$$

2. Exact solution of the method:

In two independent variables *x* and *t*, the non-linear fractional differential equation is given bellow:

$$Q\left(u, D_t^{\alpha} u, D_x^{\alpha} u, D_{tt}^{2\alpha} u, D_{xx}^{2\alpha} u, D_t^{\alpha} D_t^{\alpha} u, ...\right) = 0; 0 < \alpha < 1 \tag{2}$$

where $u = u(x,t)$ is an unaware function and α is order of the derivative. The travelling wave variables is

$$u(x,t) = U(\eta) \text{ and } \xi = ax + b\frac{t^{\alpha}}{\alpha}, \tag{3}$$

Where, *b* and *a* are constants with non-zero values. Conversion the above equations, the following ordinary differential equation (ODE) are obtained:

$$Q\left(U, U', U'', ...\right) = 0 \tag{4}$$

Where, prime $'$ is a derivative with respect to η.

Equation (4) can be revealed as follows:

$$u(x,t) = U(\eta) = a_0 + \sum_{i=1}^{m} \left(a_i \psi^i(\eta) + \frac{b_i}{\psi^i(\eta)} \right) \tag{5}$$

where $a_0, a_i, b_i, i = 1, 2, \dots \dots \dots m$ are constants to be determined; m is a positive integer that non-linear terms in the equation and $\psi(\eta)$ be satisfy the Riccati differential equation, here λ is a constant.:

$$\psi'(\eta) = \lambda + \psi^2(\eta) \tag{6}$$

In future, Riccati differential equation has some solutions. To form the general solutions of the Riccati Eq. (6).The following cases:

i) For $\lambda = 0$, then the normal solution is:

$$\psi(\eta) = -\frac{1}{\eta}.$$

ii) For $\lambda > 0$, The general trigonometric solutions of (6) are:

$$\psi(\eta) = \sqrt{-\lambda}\,\tan(\sqrt{\lambda}\eta)$$
$$\psi(\eta) = -\sqrt{\lambda}\,\cot(\sqrt{\lambda}\eta).$$

iii) For $\lambda < 0$, the general hyperbolic solutions of (6) are:

$$\psi(\eta) = -\sqrt{-\lambda}\,\tanh(\sqrt{-\lambda}\eta)$$
$$\psi(\eta) = -\sqrt{-\lambda}\,\coth(\sqrt{-\lambda}\eta).$$

From equation (5) and equation (4), obtained a polynomial in $\psi(\eta)$, collect all terms with the same power of $\psi(\eta)$ together. The coefficient of the polynomial is zero when equate it, we will get a set of over-determine algebraic equations for a_0, a_i, b_i $(i = 1, 2, \dots \dots .. m)$, and λ with the help of symbolic computation using Maple.

At last, tackling the arithmetical conditions or more potential arrangements of Riccati condition into (4), we get the arrangement of condition (2).

3. Stimulate Exact solution:

Let us suppose, the form of the conformable time fractional Hirota-Satsuma equations version of (1) are

$$\begin{cases} D_t^{\alpha} u + u_{xxx} + 6uu_x - 6vv_x = 0, \\ D_t^{\alpha} v - 2v_{xxx} - 6uvu_x = 0. \end{cases} \tag{7}$$

Let,

$$u(x,t)=U(\eta),\ \text{and}\ \eta = ax + b\frac{t^{\alpha}}{\alpha}, \tag{8}$$

Transform equation (7) into ODEs with travelling wave variable (8) for $u = U(\eta)$ and $v = V(\eta)$ as follows:

$$\begin{cases} bU' + U''' + 6UU' - 6aVV' = 0 \\ bV' - 2a^3V''' - 6aUV' = 0 \end{cases} \tag{9}$$

For homogeneous balancing between U''' and VV' and between V''' and UV' in equation (9) $m_1 + 3 = 2m_2 + 1$ and $m_2 + 3 = m_1 + m_2 + 1$ respectively.
So $m_1 = 2, m_2 = 2$.
Thus, equation (7) has a solution of the form:

$$\begin{cases} U(\eta) = a_0 + a_1\psi(\eta) + a_2\psi^2(\eta) + \dfrac{b_1}{\psi^1(\eta)} + \dfrac{b_2}{\psi^2(\eta)} \\ V(\eta) = c_0 + c_1\psi(\eta) + c_2\psi^2(\eta) + \dfrac{d_1}{\psi^1(\eta)} + \dfrac{d_2}{\psi^2(\eta)} \end{cases} \tag{10}$$

Where from equation (6)

$$\psi'(\eta) = \lambda + \psi^2(\eta)\ \text{and}\ \psi''(\eta) = 2\left(\psi(\eta)\lambda + \psi^2(\eta)\right) \tag{11}$$

Substituting equation (10) and equation (11) into equation (7) yields a set of algebraic equations for $a_0, a_1, a_2, b_1, b_2, c_0, c_1, c_2, d_1, d_2, a, b, \lambda$. These algebraic equations system are obtained as

$-48a^3c_2 - 12aa_2c_2 = 0$

$-6Aa_2c_1 - 12a^3c_1 - 12aa_1c_2 = 0$

$-80a^3c_2\lambda - 12\lambda aa_2c_2 - 6aa_1c_1 - 12aa_0c_2 + 2bc_2 = 0$

$-6\lambda aa_2c_1 - 16a^3c_1\lambda - 12\lambda aa_1c_2 - 12ab_1c_2 + 6aa_2d_1 + bc_1 - 6aa_0c_1 = 0$

$-32a^3c_2\lambda^2 - 6\lambda aa_1c_1 - 12\lambda aa_0c_2 + 2bc_2\lambda - 6Aac_1 + 12Aad_2 + 6Aad_1 - 12ab_2c_2 = 0$

$-12\lambda ab_1c_2 + 6\lambda aa_2d_1 - 6\lambda aa_0c_1 + 6aa_0d_1 - 6ab_2c_1 + 12aa_1d_2 + bc_1\lambda + 4a^3d_1\lambda - 4a^3c_1\lambda^2 - bd_1 = 0$

$-6\lambda ab_1c_1 + 12\lambda aa_2d_2 + 6\lambda aa_1d_1 - 12\lambda ab_2c_2 - 2bd_2 + 6ab_1d_1 + 32a^3d_2\lambda + 12aa_0d_2 = 0$

$6\lambda aa_0d_1 - 6\lambda ab_2c_1 + 16a^3d_1\lambda^2 + 12\lambda aa_1d_2 - bd_1\lambda + 6ab_2d_1 + 12ab_1d_2 = 0$

$-2bd_2\lambda + 6\lambda ab_1d_1 + 80a^3d_2\lambda^2 + 12\lambda aa_0d_2 + 12ab_2d_2 = 0$

$6\lambda ab_2d_1 + 12\lambda ab_1d_2 + 12a^3d_1\lambda^3 = 0$

$48a^3d_2\lambda^3 + 12\lambda ad_2b_2 = 0$

We obtain the cases bellow by using Maple system,

Case 1

$$a_0 = -\frac{1}{6}\frac{16\lambda a^3 - b}{a}, a_1 = 0, a_2 = -4a^2, b_1 = 0, b_2 = -4a^2\lambda^2,$$

$$c_0 = \frac{1}{6}\frac{(-8\lambda a + 16\lambda a^3 - b - ba)\sqrt{4a^3 - 2a}}{a(2a^2 - 1)}, c_1 = 0, c_2 = 2\sqrt{4a^3 - 2a}, d_1 = 0,$$

$$d_2 = 2\left(\sqrt{4a^3 - 2a}\right)\lambda^2$$

Therefore the solutions are:

$$u_1(x,t) = -\frac{1}{6}\frac{16\lambda a^3 - b}{a} + 4a^2\lambda tanh\left(\sqrt{-\lambda}\xi\right)^2 + \frac{4a^2\lambda}{tanh\left(\sqrt{-\lambda}\xi\right)^2} \tag{12}$$

$$v_1(x,t) = \frac{1}{6}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)\sqrt{4a^3 - 2a}}{a(2a^2 - 1)} - 2\sqrt{4a^3 - 2a}\lambda tanh\left(\sqrt{-\lambda}\xi\right)^2 - \frac{2\left(\sqrt{4a^3 - 2a}\right)\lambda}{tanh\left(\sqrt{-\lambda}\xi\right)^2} \tag{13}$$

Case 2

$a_0 = -\frac{1}{6}\frac{16\lambda a^3 - b}{a}, a_1 = 0, a_2 = -4a^2, b_1 = 0, b_2 = -4a^2\lambda^2,$

$c_0 = \frac{1}{6}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)(-\sqrt{4a^3 - 2a})}{a(2a^2 - 1)}, c_1 = 0, c_2 = 2\sqrt{4a^3 - 2a}, d_1 = 0,$

$d_2 = 2\left(-\sqrt{4a^3 - 2a}\right)\lambda^2$

Thus the solution is:

$$u_2(x,t) = -\frac{1}{6}\frac{16\lambda A^3 - B}{A} + 4A^2\lambda tanh(\xi)^2 + \frac{4A^2\lambda}{tanh\left(\sqrt{-\lambda\xi}\right)^2} \quad (14)$$

$$v_2(x,t) = \frac{1}{6}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)(-\sqrt{4a^3 - 2a})}{a(2a^2 - 1)} - 2\left(-\sqrt{4a^3 - 2a}\right)\lambda tanh\left(\sqrt{-\lambda\xi}\right)^2 - \frac{2(-\sqrt{4a^3 - 2a})\lambda}{tanh\left(\sqrt{-\lambda\xi}\right)^2} \quad (15)$$

Case 3

$a_0 = -\frac{1}{6}\frac{16\lambda a^3 - b}{a}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -4a^2\lambda^2,$

$c_0 = \frac{1}{3}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)}{\sqrt{4a^3 - 2a}},$

$c_1 = 0, c_2 = 0, d_1 = 0, d_2 = 2\left(\sqrt{4a^3 - 2a}\right)\lambda^2$

Therefore, the solution is:

$$u_3(x,t) = -\frac{1}{6}\frac{16\lambda a^3 - b}{a} + \frac{4a^2\lambda}{tanh\left(\sqrt{-\lambda\xi}\right)^2} \quad (16)$$

$$v_3(x,t) = \frac{1}{3}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)}{a(2a^2 - 1)} - \frac{2(-\sqrt{4a^3 - 2a})\lambda}{tanh\left(\sqrt{-\lambda\xi}\right)^2} \quad (17)$$

Case 4

$a_0 = -\frac{1}{6}\frac{(16\lambda a^3 - b)}{a}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -4a^2\lambda^2,$

$c_0 = \frac{1}{3}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)}{(-\sqrt{4a^3 - 2a})},$

$c_1 = 0, c_2 = 0, d_1 = 0, d_2 = 2\left(-\sqrt{4a^3 - 2a}\right)\lambda^2$

Also, the solution is:

$$u_4(x,t) = -\frac{1}{6}\frac{16\lambda a^3 - b}{a} + \frac{4a^2\lambda}{tanh\left(\sqrt{-\lambda\xi}\right)^2} \quad (18)$$

$$v_4(x,t) = -\frac{1}{3}\frac{(-8\lambda a + 16\lambda a^3 - b - ab)}{\sqrt{4a^3 - 2a}} + \frac{2(\sqrt{4a^3 - 2a})\lambda}{tanh\left(\sqrt{-\lambda\xi}\right)^2} \quad (19)$$

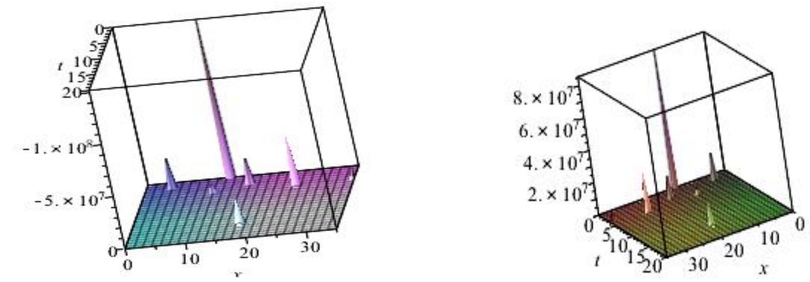
When choose $\lambda > 0$ to obtain trigonometric functions from equation (6), but when choose we use $\lambda < 0$ to obtain the solutions which having hyperbolic functions from Case 1-4 as explained in sect. 3. But if we put the value of η in the cases 1-4 is given by

$$\eta = ax + b\frac{t^\alpha}{\alpha}. \quad (20)$$

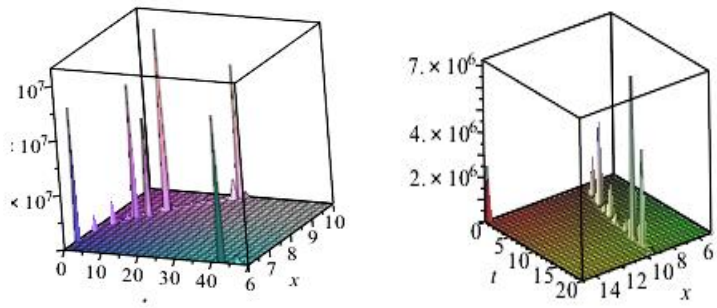
4. Graphical Represent of some obtain solutions:

Graphical representation is an important tools for information and demonstrate the solutions of the problems. Here some graphical illustrations of the conformable time Fractional on Hirota-Satsuma equations at different time levels with different values of α: $u(x,t)$ is given figure-1 their corresponding solutions of $v(x,t)$ are given in figure-2 respectively.

α = 0.1



α = 0.5



α = 1

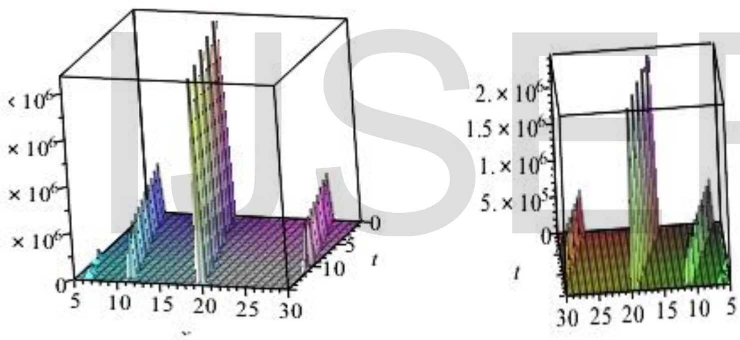


Figure: Profiles of $u(x,t)$ and $v(x,t)$ at different time levels

5. Conclusion

All in all, different exaggerated capacity answers for the time partial variation Hirota-Satsuma conditions are developed utilizing the *tanh* extension strategy in the feeling of the recently contrived comparable fragmentary subsidiary. We utilized the wave change to change the issue over to a conventional separation condition. The Maple programming is utilized for the arrangement of the arrangement of logarithmic conditions and furthermore for the graphical delineations, individually. We saw that the aftereffects of the time partial on Hirota-Satsuma conditions isolated and produce a wave setup when α is near 0 (full memory), and the design disappears when α is close to1.

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